

Evaluating and Improving Predictions

R^2 , Adding Predictors, Transformations, and Polynomials

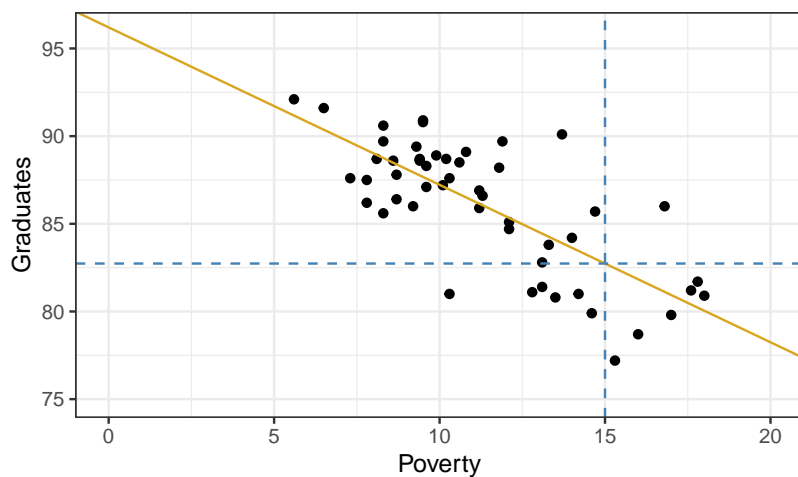
In the last lecture we built our first prediction machine: a line drawn through a scatter plot that minimizes the sum of squared residuals. In these lecture notes we focus on two questions: How can we evaluate the quality of our predictions? and How can we improve them?

Evaluating the fit to your data

Once you have fit a linear model to a scatter plot, you are able to answer questions such as:

What graduation rate would you expect for a state with a poverty rate of 15%?

Graphically, this can be done by drawing a vertical line from where the poverty rate is 15% and finding where that line intersects your linear model. If you trace from that intersection point horizontally to the y-axis, you'll find the predicted graduation rate.



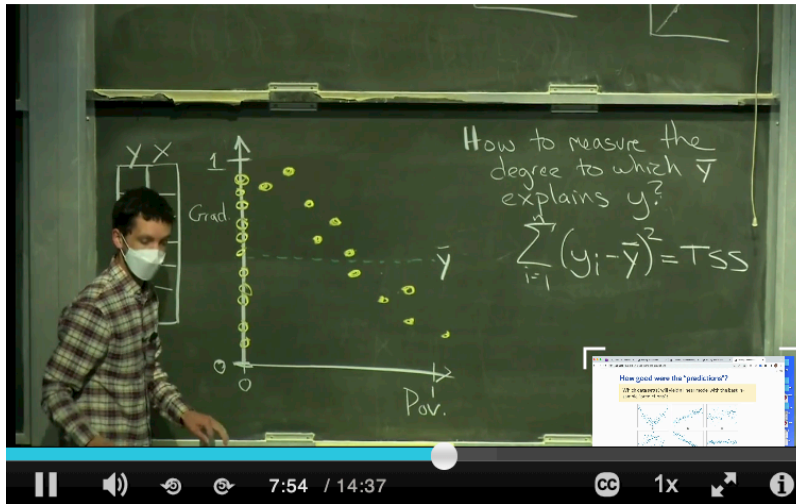
From the plot above, we can tell that the model yields a prediction around roughly 82.5%. To be more precise, we could plug the x-value into our equation for the line and solve.

$$\hat{y} = 96.2 + -0.89 \cdot 15 = 82.85$$

So how good of a prediction is 82.85%? Until we observe a state with a poverty rate of 15%, we'll never know! What we *can* know, however, is how well our model explains the structure found in the data that we *have* observed. For those observations, we have both the predicted (or fitted) values \hat{y}_i as well as their actual y-values y_i . These can be used to calculate a statistic that measures the explanatory power of our model.

Measuring explanatory power: R^2

R^2 is a statistic that captures how good the predictions from your linear model are (\hat{y}) by comparing them another even simpler model: \bar{y} . To understand how this statistic is constructed please watch this short video found in the Media Gallery on bCourses (14 minutes).



R-squared (R^2) A statistic that measures the proportion of the total variability in the y-variable (total sum of squares, TSS) that is explained away using our model involving x (sum of squares due to regression, SSR).

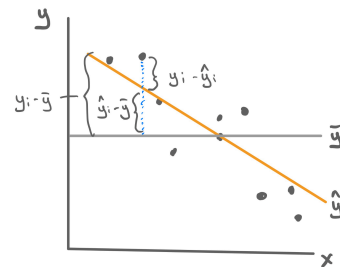
$$R^2 = \frac{SSR}{TSS} = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

Because the total variability is composed of the explained and the unexplained variability, R^2 can be equivalently formulated as 1 minus the proportion of total variability that is *unexplained* by the model, which uses the more familiar residual sum of squares (RSS).

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

R^2 has the following properties:

1. Always takes values between 0 and 1.
2. R^2 near 1 means predictions were more accurate.
3. R^2 near 0 means predictions were less accurate.



Example: Poverty and Graduation

To fit the least squares linear regression model to predict graduation rate using the poverty rate, we turn to the familiar `lm()` function.

```
m1 <- lm(Graduates ~ Poverty, data = poverty)
```

For this particular model, $R^2 = .56$. This means that poverty rate is able to explain about 56% of the variability found in graduation rates. That's a good start!

Improving predictions

R^2 allows us to quantify how well the model explains the structure found in the data set. From a model-building standpoint, it gives us a goal: to find a model with the highest possible R^2 . Here we outline three different methods for pursuing this goal - adding predictors, transformations, and polynomials - and we'd look at a different data set for each one.

Adding Predictors

Let's return to the data set that that we studying when we first learned about multiple linear regression: ratings of Italian restaurants from the ZAGAT guide. For each of the 168 restaurants in the data set, we have observations on the average price of a meal, the food quality, the quality of the decor, the quality of the service, and whether it is east or west of Fifth Avenue.

```
# A tibble: 168 x 6
  restaurant price food decor service geo
  <chr>      <dbl> <dbl> <dbl> <dbl> <chr>
1 Daniella Ristorante 43 22 18 20 west
2 Tello's Ristorante 32 20 19 19 west
3 Biricchino 34 21 13 18 west
4 Bottino 41 20 20 17 west
5 Da Umberto 54 24 19 21 west
6 Le Madri 52 22 22 21 west
```

```

7 Le Zie          34    22    16    21 west
8 Pasticcio      34    20    18    21 east
9 Belluno        39    22    19    22 east
10 Cinque Terre  44    21    17    19 east
# i 158 more rows

```

Maybe we want a model that will tell us how much we will have to spend at a new restaurant that is not upfront about its pricing; or maybe we just opened a new restaurant and want to know how much customers expect to spend. So price will serve as our response variable, leaving us four possible predictor variables. Let's fit four different regression models, each one incorporating more more information by adding a predictor.

```

m1 <- lm(price ~ food, data = zagat)
m2 <- lm(price ~ food + geo, data = zagat)
m3 <- lm(price ~ food + geo + decor, data = zagat)
m4 <- lm(price ~ food + geo + decor + service, data = zagat)

```

Unfortunately we can't visualize these four linear models as four lines on a scatterplot because only the first model describes a line. The second describes two parallel lines; the third describes two parallel planes in 3D; the fourth describes two parallel hyperplanes in 4D ().

We can, however, compare these four models in an arena where they're all on the same playing field: how well they predict price. To quantify that, we can calculate the R^2 value for each.

```

model R_squared
1    m1 0.3931835
2    m2 0.3987720
3    m3 0.6278808
4    m4 0.6278809

```

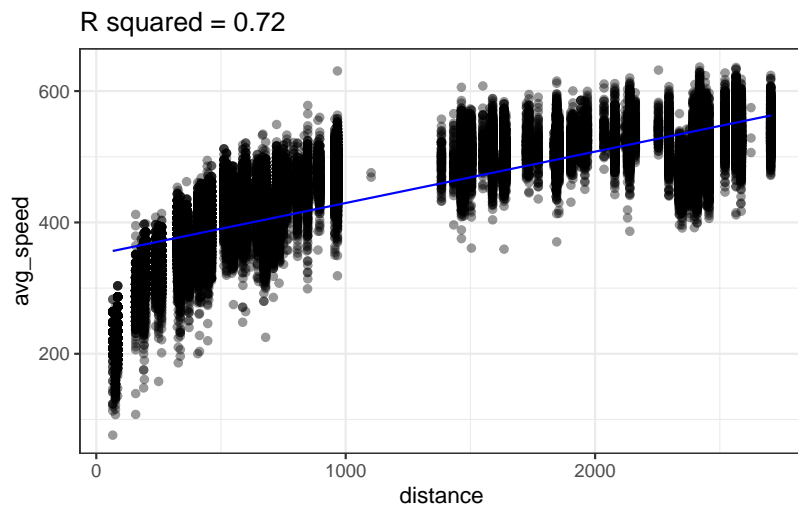
Observe that the more information we provide the model - by adding predictors - the greater the R^2 becomes! This is not a particular characteristic of the ZAGAT data set but of R^2 in general. Adding new predictors will never lower the R^2 of a model fit using least squares.

Non-linear transformation

The world is not always linear. We can create *non-linear* prediction models by building off the above *linear model* machinery. To demonstrate how to use this approach to increase the predictive power of our model, we'll turn to a non-linear trend that should look familiar...

A single non-linear term

Take a question from flights lab as an example where we plot the average airspeed vs. flight distance. First let's try fitting a linear model.



A linear model does not seem appropriate to model average speed from distance. There does appear to be a *monotonically increasing* trend, but it starts out steeper then flattens out¹. This trend is reminiscent of functions like *log* or *square root*.

Lets try transforming our predictor (distance) with the log function to create a new variable called `log_dist`.

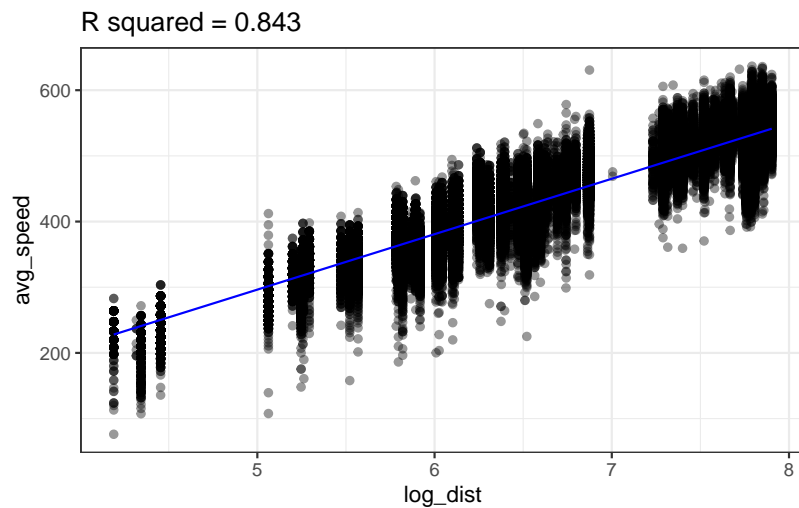
¹We call this concave or sometimes *diminishing marginal returns*.

```
flights <- flights |>
  mutate(log_dist = log(distance))
```

We can then fit a linear model using this new `log_dist` variable as the predictor.

```
lm_speed_from_log_dist <-
  lm(formula = avg_speed ~ log_dist, data=flights)
```

Looking at the data below, we see there does seem to be a linear relationship between `avg_speed` and our new variable `log_dist`! Notice the x-axis in the below plot is `log_dist` whereas it was `distance` in the above plot.



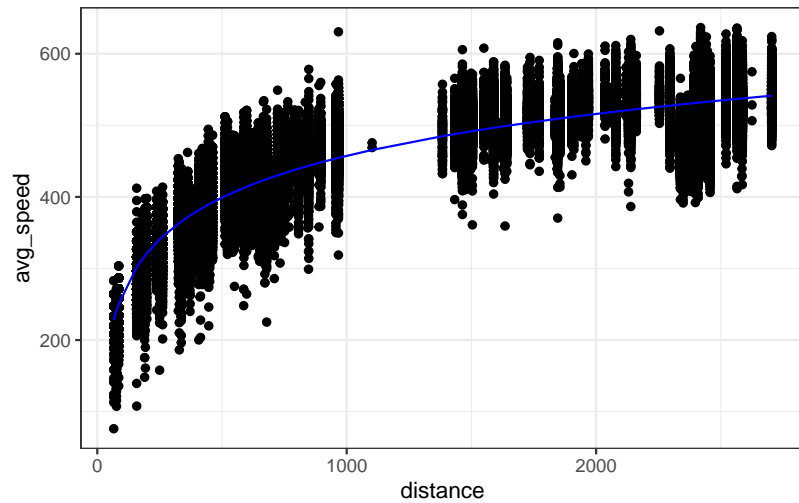
The linear model with `log_dist` ($R^2 = 0.843$) predicts `avg_speed` better than the linear model with `distance` ($R^2 = 0.72$)

We can now think of our predictive model as

$$\hat{y} = b_0 + b_1 \cdot \log(x)$$

In other words, our model is *non-linear* since x appears inside of a logarithm. We can plot the non-linear prediction function

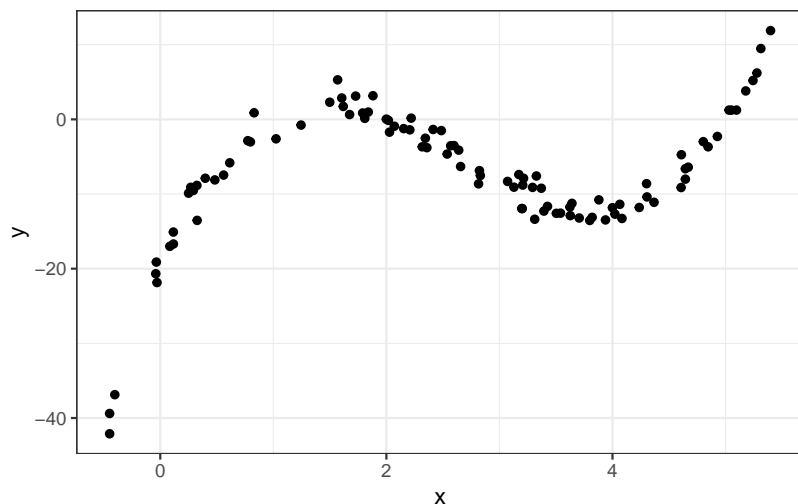
in the original predictor distance and we see the prediction function is curved!



So is this a linear model or a non-linear model? It's both. We created a new variable `log_dist` by transforming the original variable; the prediction function is a **linear** function of this new variable. But we can also think of this as a function of the original variable `distance`; the prediction function is a **non-linear** function of this original variable.

Polynomials

Sometimes we need an more complex transformation than just a simple function (e.g. \sqrt{x} , $\log(x)$, x^2 , ...). Take the following example where there is a strong association between x and y , but it's not linear (this data, admitted, was simulated in R).



So how should we model this? Polynomials to the rescue!

A **polynomial** is a function like

$$f(x) = -20 + 34x - 16x^2 + 2x^3$$

More generally a polynomial is a function like

$$f(x) = c_0 + c_1 \cdot x + c_2 \cdot x^2 + \dots + c_d \cdot x^d$$

where the $d + 1$ coefficients c_0, c_1, \dots, c_d are constants. The number d is called the *degree* of the polynomial – this is the largest exponent that appears.

Polynomials are flexible functions that can be quite useful for modeling. We can fit a polynomial model by adding new transformed variables to the data frame then fitting a linear model with these new transformed variables. This is just like how we fit a logarithmic function before by adding a new log transformed variable to the data frame then fit a linear model.

The prediction function here is a polynomial given by

$$\hat{y} = -20.086 + 34.669 \cdot x - 16.352 \cdot x^2 + 2.042 \cdot x^3$$

The Ideas in Code

Inspect model output with the broom library

Consider the code we ran earlier to fit a linear model which can predict graduation rate using the poverty rate.

```
m1 <- lm(Graduates ~ Poverty, data = poverty)
```

When you run this code, you'll see a new object appear in your environment: `m1`. This new object, though, is not a vector or a data frame. It's a much richer object called a *list* that stores all sorts of information about your linear model. You can click through the different part of `m1` in your environment pane, or you can use functions from the `broom` package to extract the important components using code.

```
library(broom)
glance(m1)
```

```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
  <dbl>      <dbl> <dbl>   <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl>
1    0.558        0.549  2.50    61.8 3.11e-10     1 -118.  242.  248.
# i 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

The `glance()` function returns a series of different metrics used to evaluate the quality of your model. First among those is r-squared. Because the output of `glance()` is just another data frame, we can extract just the r-squared column using `select()`.

```
glance(m1) |>
  select(r.squared)
```

```
# A tibble: 1 x 1
  r.squared
  <dbl>
1    0.558
```

Here's the R^2 we got earlier!

Fitting polynomials in R with `poly()`

In R, we can fit polynomials using the `poly()` function. Here is the code that was used to fit the polynomial earlier in the notes.

You do not need to worry about the meaning behind the `raw = TRUE` argument. The simulated data frame mentioned earlier is called `df`, and has two variables in it: `predictor` and `response`.

```
lm(formula = response ~ poly(x = predictor,
                             degree = 3,
                             raw = TRUE), data = df)
```

Call:

```
lm(formula = response ~ poly(x = predictor, degree = 3, raw = TRUE),
    data = df)
```

Coefficients:

```
(Intercept)
-20.086
poly(x = predictor, degree = 3, raw = TRUE)1
 34.669
poly(x = predictor, degree = 3, raw = TRUE)2
-16.352
poly(x = predictor, degree = 3, raw = TRUE)3
  2.042
```

Making predictions on a new observation with `predict()`

We have spending a lot of time talking about how to fit a model meant for predicting, but have not actually done any predicting! The `predict()` function can help us do this. It takes in two main arguments:

- **object**: This is the linear model object which contains the coefficients b_0, \dots, b_p . In the graduate and poverty example, this object was `m1`. We had `m1` through `m4` in the ZAGAT example.

- **newdata:** This is a *data frame* containing the new observation(s). This data frame must at least contain each of the predictor variables used in the column, with a value of these variables for each observation.

Example: ZAGAT food rating

Here, we will use `m2` from the ZAGAT example. This model used *food* and *geo* in an attempt to predict price at a restaurant.

First, let's make a new data frame with a couple of new, made-up observations.

```
restaurants <- data.frame(  
  food = c(25, 17),  
  geo = c("east", "west"))
```

One of these restaurants is located in east Manhattan and has a food score of 25/30, while the other one is in west Manhattan and has a food score of 17/30.

Now, we can use this data frame alongside our `m2` model object to make predictions for the prices.

```
predict(object = m2, newdata = restaurants)
```

```
      1      2  
55.89738 31.44043
```

We are predicted to have to pay roughly \$56 at the first restaurant and roughly \$31 at the second.

Summary

In this lecture we learned how to evaluate and improve our predictions. While there are many metrics to measure the explanatory power of a model, one of the most commonly used is R^2 , the proportion of the variability of the y that is explained by the model.

To improve our predictions - and increase the R^2 - we saw three different strategies. If you have additional predictors in your data frame, its easy as pie to add them to your regression model and you are guaranteed to increase your R^2 .

A second strategy is capture non-linear structure by creating new variables that are simple transformations of the existing variable. The third approach, also targeting non-linear structure, is to replace a single predictor with a polynomial.