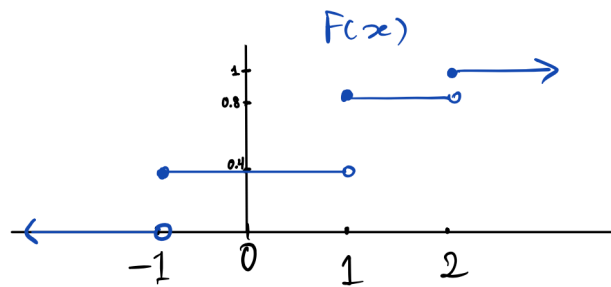


**Question 1: Expected value from cdf of  $X$**

Given the cdf of the random variable  $X$  below, compute  $E(X)$  and  $Var(X)$ . Note that:

$$F(x) = \begin{cases} 0, & x < -1 \\ 0.4, & -1 \leq x < 1 \\ 0.8, & 1 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$



**Question 2: How many red tickets**

A box of 100 tickets contains 45 red tickets, 50 blue tickets, and 5 yellow tickets. Twenty tickets will be drawn from this box at random, with replacement. What is the expected number of red tickets in the sample?

**Question 3: Expectations of linear transformations**

Let  $X$  be a random variable such that  $E(X) = 4$ . Let  $Y = 2 + 3X$ . What is  $E(Y)$ ?

**Question 4: Continuous random variables**

Consider the function  $f(x)$  where:

$$f(x) = \begin{cases} x + 1, & -1 \leq x \leq 0 \\ 1 - x, & 0 \leq x \leq 1 \\ 0, & \text{for all other } x \end{cases}$$

- (a) Does  $f$  satisfy the criteria to be a probability density function? (It should be non-negative and integrate to 1, that is, the total area under the curve is 1.)

- (b) What is your guess for  $E(X)$ , without doing any integration or summation?

**Question 5: Sums of random variables**

A discrete random variable  $X$  takes values from  $-5$  to  $5$  with  $E(X) = 0$  and  $SD(X) = 3$ . Define  $X_1, X_2, \dots, X_n$  to be independent random variables with the same distribution as  $X$ .

- (i) Does this mean (for example) that if we know that  $X = 1$ , we know that  $X_k = 1$  for all of  $X_1, X_2, \dots, X_n$ ? Why or why not?

- (ii) Let  $n = 100$  and consider  $S_{100} = X_1 + X_2 + \dots + X_{100}$ . What is the expected value and SD of  $S_{100}$ ?

- (iii) Now, let  $n = 200$ , so  $S_{200} = X_1 + X_2 + \dots + X_{200}$ . Is  $SD(S_{200}) = 2 \times SD(S_{100})$ ? Explain your answer.