Question 1 Rolling a die

(a) I roll a fair **eight-sided** die twice. What is the probability that the rolls are **different**?

(b) Now I roll the same die 4 times. What is the probability that I roll the **same** number on all 4 rolls?

Question 2 Independent? Mutually exclusive?

(a) Consider the events A and B which have probabilities 0.6 and 0.5 respectively, and we further have that $P(A \cup B) = 0.8$. Are A and B mutually exclusive, independent, both, or neither?

(b) What is the probability that *neither* A *nor* B will be true?

Question 3 Bella and her toys

My dog Bella has three toys that she loves: A stuffed and squeaky octopus, an orange ball, and a thick rope. One day we hid the octopus (too squeaky!) so she had to pick the ball or the rope. Each time she picks a toy, she chooses it independently of all the other times (like a coin toss). That day, she was busy, so went to her toys three times.

Define the events A and B where A is the event that she picked the rope *at most* one time, and B is the event that the toys she picked that day included *both* the rope and the ball.

Are A and B independent?

Question 4

Suppose A and B are non-empty events such that $P(A) = 0.5$ and $P(B) = 0.7$. What is the smallest and biggest that their union, $P(A \cup B)$, and their intersection, $P(A \cap B)$, can be?

Question 5 The Paradox of Chevalier de Méré

Antoine Gombaud, the self-styled Chevalier, lived in the 17th century and liked to gamble on the outcomes of die rolls. One game that he was fond of involved betting money on getting at least one six in 4 rolls of a fair six-sided die. We know that he computed this probability as $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6} = \frac{2}{3}$.

(a) We discussed how this is incorrect, but what is the *correct* probability of getting at least one six in 4 rolls of a fair die?

(b) Another game that he used to play was betting on rolling a pair of sixes at least once in 24 rolls of a pair of fair six-sided dice. He computed this probability as $24 \times \frac{1}{36} = \frac{2}{3}$. This is also incorrect. What is the correct answer?

Question 6 Chance of having H1N1

It was established that the H1N1 (swine flu) pandemic of 2009 originated in pigs from a very small region in central Mexico. Early cases in the US were associated with recent travel to Mexico. Say at that time, a college student went to Mexico and let's assume that at the time, the rate of infection for travelers to Mexico was 1 in 1,000. Let's suppose that the test for H1N1 has 99% accuracy. That is the chance of a false positive is 1% and the chance of a false negative is also 1%. If a person has a positive test result, what is the chance that they truly have H1N1? You can work it out using conditional probabilities and Bayes' rule, or just approximate this probability using a heuristic argument as in the slides.