Consider picking numbers from the following box.

$$
\begin{array}{|l|l|l|l|}
\hline 0 & 1 & 2 & 3 \\
\hline
\end{array}
$$

Let $A$ be the event that the first pick yields an even number; $B$ be the event that the second pick is greater than or equal to one.

1. Pick two numbers without replacement. Find $\mathrm{P}(B \mid$ first pick is 0$)$.
2. Pick two numbers without replacement. Find $\mathrm{P}(B \mid$ first pick is 2$)$.
3. Pick two numbers with replacement. Find $\mathrm{P}(B \mid A)$.

Consider a fair, eight-sided die.
4. I roll the die four times. What is the probability that I roll the same number on all four rolls?
5. I roll the die twice. What is the probability that the rolls are different?

My dog Bella has two toys that she loves: an orange ball, and a thick rope. Each time she picks out a toy, she chooses it independently of all the other times (like a coin toss). That day, she was busy, so went to her toys three times.
Define the events $A$ and $B$ where:
$A$ is the event that she picked the rope at most one time;
$B$ is the event that the toys she picked that day included both the rope and the ball.
6. Are $A$ and $B$ independent?

An American roulette wheel has 38 pockets, of which 18 are red, 18 black, and 2 are green. In each round, the wheel is spun and a white ball lands in one of these 38 pockets.
7. What is the probability of getting at the ball landing in a green pocket at least once in 5 spins of the wheel?

A European roulette wheel has 37 pockets, of which 18 are red, 18 black, and only 1 green. The roulette wheel is numbered 0 through 36 .
8. Write R code to simulate three spins of this wheel.
9. Now imagine that after each of the three spins, a pocket disappears. Simulate three spins of this magic wheel.

We will now perform our first simulation of the year! For the following questions, consider the European roulette wheel of Question 7 and ensure your Quarto document will present the same results each time it is rendered. Write your code in the spaces below.
10. Create three vectors: one which contains 100 simulated spins of the European roulette wheel (call this one_hundred), one which contains 1,000 such spins (call this one_thousand), and another which contains 10,000 such spins (call this ten_thousand).
11. Create a new vector that returns TRUE/FALSE values for each element in one_hundred, where TRUE means that the number spun is greater than 18 , and save it. Repeat these steps for the one_thousand and ten_thousand vectors.
12. Find the proportion of numbers spun in each simulation that were greater than 18 (write the code and the proportion). Hint: how can you take a proportion of a logical vector?
13. Comment on how the proportions changed with respect to the true probability of spinning a number greater than 18 as the number of spins increased.

